EXERCISE – II HINTS & SOLUTIONS

Sol.1 A,B,C

$$I = \int_0^{2\pi} \sin^2 x\,dx \quad \text{period is } \pi$$

$$= \pi \int_0^{\pi} \sin^2 x\,dx = 4\int_0^{\pi/2} \sin^2 x\,dx = 4\int_0^{\pi/2} \cos^2 x\,dx$$

Sol.2 A,B

$$I = \frac{\pi}{2}\int_0^{\pi} f(\sin x)\,dx$$

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$$I = \pi \int_0^{\pi/2} f(\sin x)\,dx$$

Sol.3 A,C

$$I = \int_0^{\infty} \frac{x}{(1+x)(1+x^2)}dx \quad ...(1)$$

Put $x = \frac{1}{4}$ $\Rightarrow$ $dx = -\frac{dt}{t^2}$

$$= -\int_{\infty}^{\infty} \frac{\frac{1}{t}}{\left(1+\frac{1}{t}\right)\left(1+\frac{1}{t^2}\right)} \cdot \frac{dt}{t^2} = \int_0^{\infty} \frac{dt}{(1+t)(1+t^2)}$$

$$I = \int_0^{\infty} \frac{dx}{(1+x)(1+x^2)} \quad ...(2)$$

(1) + (2)

$$2I = \int_0^{\infty} \frac{dx}{(1+x^2)} = \tan^{-1} x \Big|_0^{\infty}$$

$$2I = \frac{\pi}{2}$$

$$I = \frac{\pi}{4}$$

Sol.4 A,B,C,D

$$I = \int_a^b \frac{|x|}{x}dx \qquad \therefore \quad a < b$$

$x > 0$ $\quad I = \int_a^b \frac{x}{x}dx = b - a$

$x < 0$ $\quad I = -\int_a^b dx = a - b$

Sol.5 A,D

$$f(x) = \int_b^x (\cos^4 t + \sin^{-1} t)\,dt$$

$$= f(x + x) = \int_b^{\pi+x} (\cos^4 t + \sin^{-1} t)\,dt$$

$$= f(x) + \int_b^x (\cos^4 t + \cos^{-1} t)\,dt$$

$= f(x) + f(x)$

$$= f(x) + \int_0^{\pi} (\cos^4 t + \cos^4 t)\,dt$$

$$= f(x) + \frac{1}{2}\int_0^{\pi/2} (\cos^4 t + \sin^4 t)\,dt$$

Sol.6 A,D

$$I = \int_0^1 \frac{2x^2 + 3x + 3}{(x+1)(x^2+2x+2)}\,dx$$

$$= \int_0^1 \frac{(x^2+2x+2) + (x^2+x) + 1}{(x+1)(x^2+2x+2)}\,dx$$

$$= \int_0^1 \frac{dx}{x+1} + \int_0^1 \frac{x}{x+2x+2}dx + \underbrace{\int_0^1 \frac{dx}{(x+1)(x^2+2x+2)}dx}_{I_3}$$

$$= \ell n\,(x+1)|_0^1 + \int_0^1 \frac{x}{x^2+2x+2}dx + I_3$$

$$= \ell n2 + \frac{1}{2}\int_0^1 \frac{2x+2-2}{x^2+2x+2}\,dx + I_3$$

$$I = \ell n2 + \frac{1}{2}\int_0^1 \frac{2x+2}{x^2+2x+2} - \int_0^1 \frac{dx}{x^2+2x+2} + I_3$$

$$= \ell n2 + \frac{1}{2}\,\ell n\,(x^2+2x+2)|_0^1 - \int_0^1 \frac{dx}{1+(x+1)^2} + I_3$$

$$= \ell n2 + \frac{1}{2}\,\ell n\,5 - \frac{1}{2}\ell n2 - \tan^{-1}\,(x+1)|_0^1 + I_3$$

$$= \frac{1}{2}\,\ell n2 + \frac{1}{2}\ell n5 - \tan^{-1}2 + \tan^{-1}1 + I_3$$

$$= \frac{1}{2}\,\ell n10 - \tan^{-1}2 + \frac{\pi}{4} + I_3$$

$$I_3 = \int_0^1 \frac{dx}{(x+1)[(x+1)^2+1]}$$

$$x + 1 = \frac{1}{t}$$

$$dx = -\frac{dt}{t^2} \qquad = \int_1^{\frac{1}{2}} \frac{1}{\frac{1}{t}\left(1+\frac{1}{t^2}\right)}\left(\frac{-dt}{t^2}\right)$$

$$= -\int_1^{\frac{1}{2}} \frac{t\,dt}{t^2+1}\,dt = \frac{1}{2}\ln\,(t^2+1)|_{1/2}^1$$

$$= \frac{1}{2}\,\ell n2 - \frac{1}{2}\,\ell n\left(\frac{3}{4}\right) = \frac{1}{2}\ell n\,2 - \frac{1}{2}\ell n5 + \ell n\,2$$

$$I = \frac{1}{2}\ell n2 + \frac{1}{2}\,\ell n5 - \tan^{-1}2 + \frac{\pi}{4}$$

$$+ \frac{1}{2}\,\ell n2 - \frac{1}{2}\ell n5 + \ell n2$$

$$I = 2\,\ell n2 - \tan^{-1}2 + \frac{\pi}{4}$$

$$I = 2\,\ell n2 - \frac{\pi}{2} + \cot^{-1}2 + \frac{\pi}{4}$$

$$= -\frac{\pi}{4} + \ell n4 + \cot^{-1}2$$

Sol.7 **C,D**

$f'(\sin^2 x) = \cos^2 x$

Assume $\sin^2 x \to t$

$f'(t) = 1 - t$

$$f(t) = t - t\frac{t^2}{2} + c$$

$$1 = 1 - \frac{1}{2} + c$$

$$c = \frac{1}{2}$$

$$f(t) = t - \frac{t^2}{2} + \frac{1}{2}$$

$$f(x) = x - \frac{x^2}{2} + \frac{1}{2}$$

Sol.8 **A,B**

$$I_n = \int_0^1 \frac{dx}{(1+x^2)^n} \quad x = \tan\theta$$

$$dx = \sec^2\theta\,d\theta$$

$$I_2 = \int_0^1 \frac{dx}{(1+x^2)^2} = \int_0^{\pi/4} \frac{\sec^2\theta d\theta}{\sec^4\theta} = \int_0^{\pi/4} \cos^2\theta\,d\theta$$

$$= \int_0^{\pi/4} \left(\frac{1+\cos 2\theta}{2}\right) d\theta$$

$$= \frac{1}{2}\left(\frac{\pi}{4}\right) + \frac{1}{2}\int_0^{\pi/4} \cos 2\theta \, d\theta = \frac{\pi}{8} + \frac{1}{4}$$

Sol.9 B,C

$$I = \int_1^2 f(x)\,dx$$

(A) $\lim\limits_{n\to\infty} \frac{1}{n} \sum\limits_{r=1}^{n} + \left(\frac{r}{n}\right) \to \int_0^1 f(x)\,dx$

(B) $\lim\limits_{n\to\infty} \frac{1}{n} \sum\limits_{r=n+1}^{2n} \left(\frac{r}{n}\right) \to \int_1^2 f(x)dx$

(C) $\lim\limits_{n\to\infty} \frac{1}{n} \sum\limits_{r=1}^{n} + \left(1+\frac{r}{n}\right) \to \int_0^1 f(1+x)dx$

Put 1 + x = t

$$\int_1^2 f(t)\,dt$$

(D) $\lim\limits_{n\to\infty} \frac{1}{n} \sum\limits_{r=1}^{2n} f\left(\frac{r}{n}\right) \to \int_0^2 f(x)\,dx \to \int_0^2 d(x)\,dx$

Sol.10 A,B,C,D

$f(x) = f^{\{x\}}$

{x} is perido with = 1

(A)

(B) $\int_0^1 2^{\{x\}}dx = \int_0^1 2^x dx = \left.\frac{2^x}{\ell n2}\right|_0^1 = \frac{1}{\ell n2}$

(C) $\int_0^1 2^{\{x\}}dx = \frac{1}{\ell n2} = \frac{\ell n_e \rho}{\ell ne^2}$

(D) $\int_0^{100} 2^{\{x\}}dx = 100 \int_0^1 2^{\{x\}}dx = 100\ \ell n_2 \rho$

Sol.11 A,B

$$f(x) = \int_0^x (2\cos^2 3t + 3\sin^2 3t)dt$$

$$= \int_0^x (2 + \sin^2 3t)dt$$

$$f(x+\pi) = \int_0^x (2+\sin^2 3t)dt + \int_x^{x+\pi} (2+\sin^2 3t)dt$$

$$= f(x) + \int_x^x (2+\sin^2 3t)dt$$

$$= f(x) + f(x)$$

$$f(x) = \int_0^x (2+\sin^2 3t)dt$$

$$= 2\int_0^{\pi/2} (2+\sin^2 3t)dt$$

$$= 2f\left(\frac{\pi}{2}\right)$$

$$f(x+\pi) = 2f\left(\frac{\pi}{2}\right) + f(x)$$